Implicit derivatives

Calculate the partial derivatives of the following implicitly defined functions if $z = f(x, y)$.

1. $x + y + z = \sin(yz)$
2. $x + 3y + 2z - \ln(z) = 0$
3. $x^3 + 2y^3 + z^3 - 3xyz - 2y = -3$

Solutions

To implicitly differentiate, first completely differentiate the expression:

$$f(x, y, z) = 0$$

$$f'_x\,dx + f'_y\,dy + f'_z\,dz = 0$$

If we want to find the derivative of z'_x, then y is held constant:

$$f'_x\,dx + f'_z\,dz = 0$$

We solve for:

$$\frac{dz}{dx} = -\frac{f'_x}{f'_z}$$

The same applies if we want to find z'_y, then x is held constant:

$$f'_y\,dy + f'_z\,dz = 0$$

And we solve for:

$$\frac{dz}{dy} = -\frac{f'_y}{f'_z}$$

The same result can be obtained by thinking about the chain rule, differentiating the expression $f(x, y, z)$ with respect to x:

$$f'_x + f'_z \cdot z'_x = 0$$

Since y does not depend on x, the result y'_x equals 0. We solve for:

$$z'_x = -\frac{f'_x}{f'_z}$$

1. First, we rearrange:

   $$x + y + z - \sin(xyz) = 0$$

   We calculate:

   $$z'_x = -\frac{f'_x}{f'_z}$$

   $$z'_x = -\frac{1 - \cos(xyz)yz}{1 - \cos(xyz)yx}$$

   In the other case:

   $$z'_y = -\frac{f'_y}{f'_z}$$

   $$z'_y = -\frac{1 - \cos(xyz)zx}{1 - \cos(xyz)yx}$$

2. We follow the same procedure:

   $$z'_x = -\frac{1}{2 - 1/z}$$

   $$z'_y = -\frac{3}{2 - 1/z}$$

3. $$z'_x = -\frac{3x^2 - 3yz}{3z^2 - 3xy}$$

   $$z'_y = -\frac{6y^2 - 3xz - 2}{3z^2 - 3xy}$$